Subspace Identification Method for Simulation of Closed-Loop Systems with Time Delays

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Pade approximations of time delays are usually used to simulate continuous-time closed-loop systems with time delays. When large dimensional multivariable control systems are simulated, however, the number of states can be too large to handle effectively. For example, if the 5/5 Pade approximation is used for a 10×10 process whose 100 transfer function elements are all first-order plus time delay models, 600 states plus controller states must be integrated. Simulation routines in the Control System Toolbox of MATLAB cannot be used for this problem because they are based on the matrix exponential of a very large system matrix whose dimension is larger than 600×600. The state space model from the transfer function model is not easily realized. Discretization with a zero-order hold can also be used to simulate a continuous-time, closed-loop system, but it suffers from the same dimensionality problem when a smaller sampling time is used for a better approximation. Alternatively, a subspace identification method in the frequency domain can be used to reduce the dimensionality of the problem. Subspace identification provides the proper system order automatically to simulate closed-loop systems with time delays with minimal user specifications. This method provides accurate time responses and can be used as a standard simulation method for large dimensional closed-loop systems with time delays, replacing the Pade approximation method.

Simulation of Closed-Loop Systems with Time Delays

Consider the closed-loop control system with process transfer function $G(s)$, controller $C(s)$, set point variable $R(s)$ and controlled variable $Y(s)$. The closed-loop transfer function is

$$Y(s) = (I + G(s)C(s))^{-1} G(s)C(s)R(s)$$
$$\equiv H(s)R(s)$$

To simulate the system for a certain set point change, $G(s)$ and $C(s)$ should be realized to state space models. When $G(s)$ contains time delays, individual transfer functions should be converted with Pade approximations before realization. This Pade approximation increases the number of states, which may be too large to handle for large dimensional processes. To resolve this problem, we propose a simulation method which uses the frequency response data of $H(j\omega)$. The proposed procedure is as follows:

Step 1. Calculate the frequency response data $h_{ij}(j\omega)$.

Step 2. Identify $h_{ij}(s)$ from $h_{ij}(j\omega)$ using the subspace identification method of McKelvey et al. (1996).

Step 3. Simulate the system $yi(s = h_{ij}(s)r_j(s)$.

Subspace Identification Method in the Frequency Domain

A basic subspace identification method in the frequency domain (McKelvey et al., 1996) which is a counterpart of the time domain subspace identification method (Overschee and De Moor, 1994) is summarized for the SISO case as follows.

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Step 1. From a given frequency response data set, $\{h(j\omega_i), i = 1, 2, \ldots, M\}$, calculate matrices

$$Z = \frac{1}{\sqrt{M}} \begin{bmatrix} h_1 & h_2 & \cdots & h_M \\ \exp(j\tilde{\omega}_1) h_1 & \exp(j\tilde{\omega}_2) h_2 & \cdots & \exp(j\tilde{\omega}_M) h_M \\ \cdots & \cdots & \cdots & \cdots \\ \exp(j(q-1)\tilde{\omega}_1) h_1 & \exp(j(q-1)\tilde{\omega}_2) h_2 & \cdots & \exp(j(q-1)\tilde{\omega}_M) h_M \end{bmatrix}$$

$$W = \frac{1}{\sqrt{M}} \begin{bmatrix} 1 & 1 & \cdots & 1 \\ \exp(j\tilde{\omega}_1) & \exp(j\tilde{\omega}_2) & \cdots & \exp(j\tilde{\omega}_M) \\ \cdots & \cdots & \cdots & \cdots \\ \exp(j(q-1)\tilde{\omega}_1) & \exp(j(q-1)\tilde{\omega}_2) & \cdots & \exp(j(q-1)\tilde{\omega}_M) \end{bmatrix}$$

where q is a constant, $h_i = h(j\omega_i)$ and $\tilde{\omega}_i = 2\arctan(\omega_i T/2)$.

Step 2. Calculate the QR factorization

$$\begin{bmatrix} Re(W) & Im(W) \\ Re(Z) & Im(Z) \end{bmatrix} = \begin{bmatrix} R_{11} & 0 \\ R_{21} & R_{22} \end{bmatrix} \begin{bmatrix} Q_1^T \\ Q_2^T \end{bmatrix}$$

Step 3. Calculate the SVD of R_{22} and determine the system order n by inspecting the singular values and partition the SVD such that Σ_s contains the n largest singular values as

$$R_{22} = [U_s \ U_o] \begin{bmatrix} \Sigma_s & 0 \\ 0 & \Sigma_o \end{bmatrix} \begin{bmatrix} V_s^T \\ V_o^T \end{bmatrix}$$

Step 4. Determine the system matrices

$$A_d = \left(U_s^T J_1^T J_1 U_s\right)^{-1} U_s^T J_1^T J_2 U_s$$
$$c_d = J_3 U_s$$

where

$$J_1 = \left[I_{(q-1)} \ 0_{(q-1)\times 1} \right], \ J_2 = \left[0_{(q-1)\times 1} \ I_{(q-1)} \right] \text{ and } J_3 = \left[1 \ 0_{1\times(q-1)} \right].$$

Step 5. Calculate

$$\begin{bmatrix} d_d \\ b_d \end{bmatrix} = (X^T X)^{-1} X^T \begin{bmatrix} h_1 \\ h_2 \\ \cdots \\ h_M \end{bmatrix},$$

$$X = \begin{bmatrix} 1 & c_d\left(\exp(j\tilde{\omega}_1) I - A_d\right)^{-1} \\ 1 & c_d\left(\exp(j\tilde{\omega}_2) I - A_d\right)^{-1} \\ \cdots & \cdots \\ 1 & c_d\left(\exp(j\tilde{\omega}_M) I - A_d\right)^{-1} \end{bmatrix}$$

Step 6. The estimated transfer function becomes

$$h(s) = c(sI - A)^{-1} b + d$$

where

$$A = \frac{2}{T}(I + A_d)^{-1}(A_d - I)$$

$$b = \frac{2}{\sqrt{T}}(I + A_d)^{-1} b_d$$

$$c = \frac{2}{\sqrt{T}} c_d (I + A_d)^{-1}$$

$$d = d_d - c_d (I + A_d)^{-1} b_d$$

This subspace identification method is noniterative and provides exact models for finite-dimensional rational transfer function processes (McKelvey et al., 1996).

Practical Considerations

There are several parameters to be selected for the subspace identification. The parameter T is introduced to use the discrete-time subspace identification method for identifying continuous-time models, as a sort of sampling period (McKelvey et al., 1996). We can freely specify it under the constraint that $2/T$ is not a pole of the continuous-time system. We set $T = 2\pi/\omega_{[M/2]}$. In Step 3 of the subspace identification method, a large system order n will increase the approximation accuracy, but is apt to cause numerical instability. Here, n is chosen such that the $(n+1)$-th singular value decreases by 0.001 times of the largest singular value. The parameter q limits n by $q \geq n$. Here, we set $q = 32$.

The range of frequencies greatly affects the identification performance. Before applying the subspace identification method, the Bode plot should be checked whether the frequency range spans all important frequencies with the number of frequency response data points between 100 and 500.

An explicit time delay is very hard to approximate with a rational transfer function model. Here it is extracted before applying the subspace identification. In case that $h_{ij}(j\omega) = \hat{h}_{ij}(j\omega)\exp(-j\theta\omega)$ and θ is known in advance, we may identify $\hat{h}_{ij}(s)$ for the data set of $h_{ij}(j\omega)\exp(j\theta\omega)$. In the simulation of multivariable control systems with time delays, θ can be found easily with simple calculations.

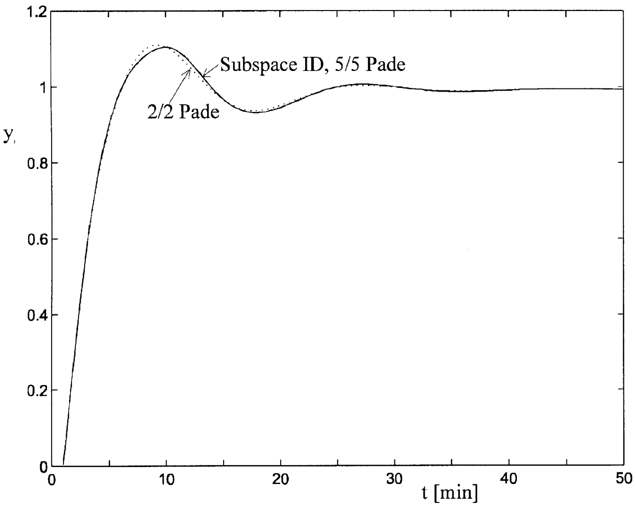

Figure 1. Output y_1 for the set point change of r_1 in Example 1.

Solid line: proposed subspace identification method; dotted line: 2/2 Pade approximation method; dashed line: 5/5 Pade approximation method.

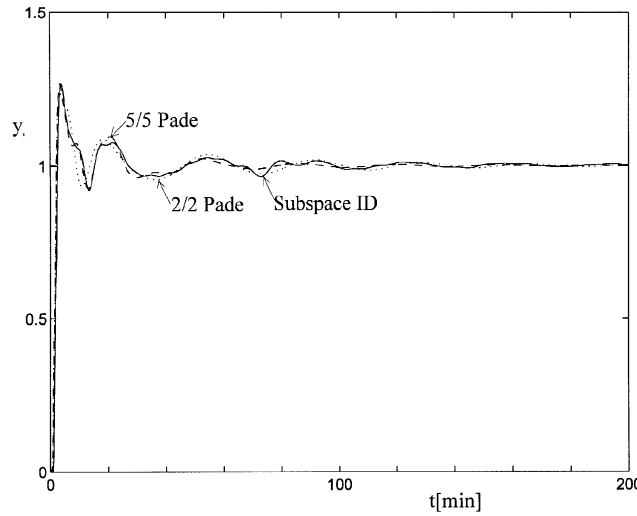

Figure 2. Output y_1 for the set point change of r_1 in Example 2.

Solid line: proposed subspace identification method; dotted line: 2/2 Pade approximation method; dashed line: 5/5 Pade approximation.

Applying this SISO simulation method to elements of $H(s)$, we can obtain time responses between any pair of set point and output variables.

Example 1

Consider the Wood and Berry column with a multiloop PI controller based on BLT tuning (Luyben, 1986)

$$G(s)=\begin{bmatrix} \dfrac{12.8\exp(-s)}{16.7s+1} & \dfrac{-18.9\exp(-3s)}{21s+1} \\ \dfrac{6.6\exp(-7s)}{10.9s+1} & \dfrac{-19.4\exp(-3s)}{14.4s+1} \end{bmatrix}$$

$$C(s)=\mathrm{diag}\left\{0.375\left(1+\frac{1}{8.29s}\right), -0.075\left(1+\frac{1}{23.6s}\right)\right\}$$

The output y_1 is simulated for the step change in r_1. The proposed method is compared with the Pade approximation method. For the subspace identification method, 200 frequency response data are used that are logarithmically equally spaced between 0.01 and 10. Because both outputs do not respond until 1 min after changes of the first manipulated variable, we can have the explicit time delay of 1 min. The time delay term is excluded in applying the subspace identification and added later for simulation of the time-domain response. The system order is calculated to be $n=9$. Figure 1 shows the simulation results. We can see that responses of the 5/5 Pade approximation and the proposed subspace identification method are not differentiated. The 2/2 Pade approximation which requires integration of 14 state equations is insufficient compared to the 5/5 Pade approximation (requires integration of 26 state equations). The proposed subspace identification method requires integration of nine state equations. The relative sum of squared errors (Table 1) shows that the proposed method is accurate enough and can be used for the purpose of simulation, replacing the Pade approximation method.

Table 1. Relative SSE of Output y_1 for the Set Point Change of r_1 in Example 1

Method	Number of State Equations	Relative SSE
1/1 Pade	11	1
2/2 Pade	15	0.0728
3/3 Pade	19	0.0090
4/4 Pade	23	0.0017
5/5 Pade	28	0.0003
Proposed	9	0.0004

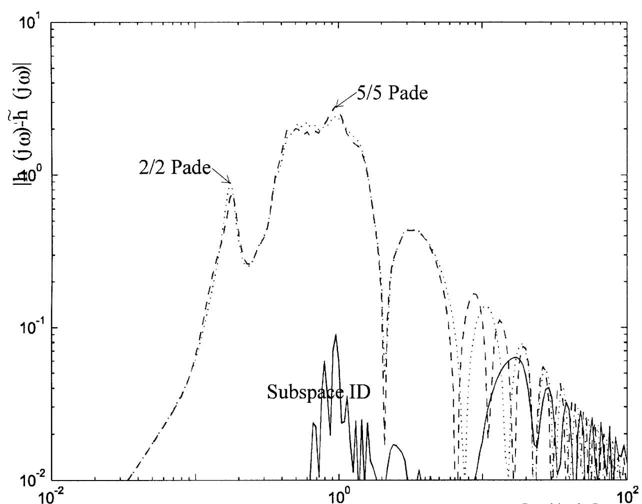

Figure 3. Absolute errors between $h_{11}(j\omega)$ and its approximations, $(|h_{11}(j\omega)-\tilde{h}_{11}(j\omega)|)$.

Solid line: proposed subspace identification method; dotted line: 2/2 Pade approximation method; dashed line: 5/5 Pade approximation.

Example 2

Consider the T4 column (Luyben, 1986) with a multiloop PI controller

$$G(s)=\begin{bmatrix} \dfrac{-1.986\exp(-0.71s)}{66.67s+1} & \dfrac{5.24\exp(-60s)}{400s+1} & \dfrac{-5.984\exp(-2.24s)}{14.29s+1} \\ \dfrac{0.0204\exp(-0.59s)}{(7.14s+1)^2} & \dfrac{-0.33\exp(-0.68s)}{(2.38s+1)^2} & \dfrac{2.38\exp(-0.42s)}{(1.43s+1)^2} \\ \dfrac{0.374\exp(-7.75s)}{22.22s+1} & \dfrac{-11.3\exp(-3.79s)}{(21.74s+1)^2} & \dfrac{-9.811\exp(-1.59s)}{11.36s+1} \end{bmatrix}$$

$$C(s)=\mathrm{diag}\left\{-\frac{24.9s+4.40}{s},\ -\frac{6.08s+0.507}{s},\ -\frac{0.303s+0.0247}{s}\right\}$$

The output y_1 is simulated for the step change in r_1. For the subspace identification method, 200 frequency response data with logarithmically equal spacing between 0.01 and 100 are used. Extraction of the explicit time delay is not applied here because the explicit time delay is small compared to the time constant. The system orders are 31 for 2/2 Pade approximation, 58 for 5/5 Pade approximation, and 31 for the proposed subspace identification method. Figure 2 shows the simulation results. There are considerable differences between responses of the 5/5 Pade approximation and the proposed subspace identification method. For the system, it is hard to obtain accurate time-domain responses. Hence, to check which one is better, we compare frequency responses. Figure 3 shows absolute errors between the exact $h_{11}(j\omega)$ and its approximations $\tilde{h}_{11}(j\omega)$, ($|h_{11}(j\omega)-\tilde{h}_{11}(j\omega)|$). We can see that the proposed subspace identification method has smaller errors than the 2/2 and 5/5 Pade approximations. Incrementing the Pade approximation order did not work well and approximation orders higher than the 9/9 Pade approximation result in unstable systems due to numerical problems.

Conclusion

We show that subspace identification in the frequency domain can be effectively used to simulate the time-domain response of large dimensional closed-loop systems with time delays. The proposed method can also be applied to large dimensional discrete-time systems.

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